Reliability assessment on maximum crack width of GFRP-reinforced concrete beams

Z. He and F. Qiu
College of Resources and Civil Engineering, Northeastern University, Shenyang, 110004, China

ABSTRACT: The suggested provisions in guideline ACI 440.1R-06 for predicting the maximum crack width of GFRP-reinforced concrete beams are assessed from the probabilistic point of view by the Rackwitz-Fiessler method. The assessment indicates that the global average reliability index is 0.57, meeting the target reliability index of serviceability ultimate state. Effective-height-to-height ratio, d/h, width, b, and height-to-width ratio, h/b, are the first three dominating influencing factors on reliability level. The average reliability index is basically independent of relative FRP reinforcement ratio, η. The effect of load effect ratio, α, on reliability level is also significant and the index tends to converge at β=0.58. The parametric study on the bond-dependent coefficient, k_b, indicates that its mean does not put any effect on reliability level while its C.O.V. is rather significant. As the C.O.V. of k_b increases from 0.05 to 0.20, the average index decreases from 0.53 to 0.41. The limit of the maximum crack width, [w], is shown to be no relations with reliability level. Specification on the limit of maximum allowable crack width is dependent only on the consideration of durability and visible tolerance.

1 INTRODUCTION

The relatively low moduli of elasticity of FRP bars can cause considerable deflection and crack width in flexural FRP-reinforced concrete beams even in their serviceability states. Many efforts have been devoted to develop the prediction models for those states. Such models become more and more accurate with expanding experimental database. But, most of those models have been modified from those corresponding to steel reinforced concrete structures, some possible potential risks should be identified carefully because of the sharp differences in mechanical properties between reinforcing bars and FRP materials (He & Ou, 2005). In 1995, Plevris *et al* (Plevris *et al*., 1995) introduced the concept of reliability into the flexural capacity design of concrete beams strengthened with CFRP laminates. After this, the flexural capacity of FRP-reinforced concrete beams was assessed from the probabilistic point of view by some researchers (Huang, 2005, He *et al*., 2006). He and Qiu (He & Qiu, 2007) conducted the reliability assessment on the serviceability state of FRP-reinforced concrete beams. In the assessment, the deflection design provisions in guideline ACI 440.1R-06 (ACI 440, 2006) were evaluated. As the result of the assessment, the deflection limit of l/120 is suggested for the short-term deflection limit for GFRP-reinforced concrete beams. As accompanying research work, this paper presents a reliability assessment on the provisions for predicting the maximum crack width of FRP-reinforced concrete beams designed by ACI 440.1R-06 (ACI 440, 2006).

2 LIMIT STATE FUNCTION

The maximum crack width, w, for FRP-reinforced concrete members is determined by the following equations,

$$w = 2\beta k_b \frac{f_f}{E_f}\sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \tag{1}$$

$$\beta = (h - kd)/d(1-k) \tag{2}$$

$$f_f = \frac{M_a}{A_f d\left(1 - k/3\right)} \tag{3}$$

$$k = \sqrt{2\rho_f n_f + \left(\rho_f n_f\right)^2} - \rho_f n_f \tag{4}$$

where, E_f is the modulus of elasticity of FRP bar; d_c is the thickness of cover from tension face to center of closest bar, d_c=h-d; h and d are the height and the effective height of cross section, respectively; s is bar spacing, s=b-2d_c; b is the width of cross section; k_b is the bond-dependent coefficient, for the case where k_b is not known from experimental data, is that a conservative value of 1.4 should be assumed; β is the ratio of distance between neutral axis and tension face to distance between neutral axis and centroid of reinforcement; f_f is the reinforcement stress of FRP bar; M_a is applied moment, it consists of the moment caused by dead load, M_G, and the moment caused by service live load, M_Q, i.e. M_a=M_G+M_Q; kd is neutral axis depth; ρ_f is reinforcement ratio, ρ_f=A_f/bd; A_f is the sectional area of FRP bar; n_f is the modular ratio of FRP bar to concrete, n_f=E_f/E_c. E_c is calculated by (ACI 318-05, 2005).

$$E_c = 4750\sqrt{f_c'} \tag{5}$$

where, f_c' is the compressive cylinder strength of concrete.

The limit state function, z, of flexural FRP-reinforced concrete beams is expressed as

$$z = \left[w\right] - \alpha_w w \tag{6}$$

where, [w] is the limit of maximum allowable crack width. The value of [w] is implicitly suggested as 0.5mm for most cases for exterior exposure and as 0.7mm for interior exposure when FRP reinforcement is used; α_w is the uncertainty factor associated with the predictions by Eq.(1). It is defined as the ratio of the actual maximum crack width of FRP-reinforced concrete beam to the predicted maximum crack width by Eq.(1). Obviously, α_w is a random variable. Through a statistical analysis of a total of 50 effective of GFRP-reinforced concrete beams (Theriault & Benmokrane, 1998; Masmoudi *et al*., 1998; Xue & Kang, 1999; Gao *et al*., 2001; Wang & Shi, 2002; Toutanji & Saafi, 2003; Liu, 2003; Wang & Belarbi, 2005), the statistical mean and the standard deviation of α_w are 0.911 and 0.139, respectively. For simplicity, the normal distribution is assumed for α_w and k_b in the assessment.

3 RELIABILITY ANALYSIS

3.1 *Design variables*

From Eqs.(1) to (6), we can observe that the reliability level of maximum crack width for FRP-reinforced concrete beams are associated with many factors, e.g., sectional width, b, sectional height, h, effective height, d, the modulus of elasticity of concrete, E_c, and FRP bar, E_f, FRP reinforcement ratio, ρ_f, load effects, M_G and M_Q, etc. Among these factors, b, h, d, E_c and E_f are treated as random design variables. The statistics of b, h and d are selected from Ellingwood *et al*.'s report (Ellingwood *et al*., 1980). As we know, the modulus of elasticity of concrete, E_c, is closely associated with concrete strength, f_c', so Eq.(5) can be used to evaluate the statistical data of E_c by the statistical model of f_c' in Nowak & Szerszen's statistical data (Nowak & Szerszen, 2003). The same method is also applied to evaluate the statistics of E_f through linear regression of 50 data points between E_f and GFRP tensile strength, f_{fu}, as expressed in Eq. (7) (Qiu, 2007).

$$E_f = 28.45 + 19 f_{fu} \text{ (GPa)} \tag{7}$$

The statistical data of GFRP bar in Group A and Group B are abstracted from Pilakoutas *et al*'s paper (Pilakoutas *et al*., 2002) and He's report (He, 2003), respectively. The statistical data of dead load and live load are listed in Table 3 (Ellingwood *et al*., 1980). ρ_f is treated as a deterministic variable. To make the assessment more general, two extreme nominal values (A and B) are selected for each random design variable as well as twenty five FRP relative reinforcement ratios, η=ρ_f/ρ_{fb}, where ρ_{fb} is the balanced FRP reinforcement ratio as specified in ACI

440.1R-06 (ACI 440, 2006). Thus, it defines a broad design space with a total of 2^5×25=800 design cases. The Rackwitz-Fiessler method (Rackwitz & Fiessler, 1978) is applied to calculate reliability index. Five different load effect ratios, i.e. $\alpha=M_Q/M_G$=0.5, 1.0, 1.5, 2.0 and 2.5, are selected to evaluate the effect of α upon reliability level. So, there are 800×5=4,000 indexes. Assume that both M_Q and M_G are caused by uniform distribution loads in the reliability analysis. k_b is taken as 1.40 in the analysis.

Table 1 Statistical data of design variables

Design variable	Nominal (Group A)	Mean μ, Standard deviation, σ	Nominal (Group B)	Mean μ, Standard deviation, σ	Probability distribution
b (mm)	200	$\mu=b$+2.54 σ=3.66	500	$\mu=b$+2.54 σ=3.66	Normal
h (mm)	1.5b	$\mu=h$-3.05 σ=6.35	3.0b	$\mu=h$-3.05 σ=6.35	Normal
d (mm)	0.8h	$\mu=d$-4.70 σ=12.70	0.95h	$\mu=d$-4.70 σ=12.70	Normal
E_c (MPa)	21.60	μ=25.12 σ=1.17	30.54	μ=32.27 σ=0.576	Normal
E_f (GPa)	41.83	μ=43.84 σ=0.77	49.08	μ=51.65 σ=0.86	Normal

Table 2 Statistical data of dead load and live load

Load pattern	Mean/Nominal, κ	C.O.V.[a], δ	Probabilistic distribution	Load factor
Dead	1.05	0.10	Normal	1.0
Live[b]	1.00	0.25	Extreme I	1.0

[a]: C.O.V.= coefficient of variance. [b]: 50-year maximum

3.2 *Effects of design variables*

Figure 1 shows the effects of all design variables associated with sectional resistance on average reliability level of crack width of GFRP-reinforced concrete beams. The average reliability index for each design variable is obtained by dividing all the indexes into two groups according to the nominal value of the variable considered and then to calculate the average index in each group. It can be seen that the effect of effective-height-to-height ratio, d/h, is the most significant influencing factor, accompanied by width, b, and height-to-width ratio, h/b. As d/h increases from 0.8 to 0.95, the average reliability index increases up to 31%. A change of b from 200mm to 500mm could result in an increase of 26.8% in average reliability level. If h/b increases from 1.5 to 3.0, the average index would increase up to 10%. The effects of the modulus of elasticity of concrete and GFRP bar, i.e. E_c and E_f, are relatively insignificant in contrast with d/h, b and h/b. A decrease of 4.4% in average reliability index could be expected if E_f increases from 41.83GPa to 49.08GPa. As for E_c, the decrease would be 2.3% if it increases from 21.60GPa to 30.54GPa. The horizontal straight line in Figure 1 shows global the average reliability index of all indexes. As suggested in the *General principles on reliability for structures* (ISO/DIS 2394, 1998) for the target reliability index of serviceability ultimate state, the target reliability index could be 0 for reversible limit state and 1.5 for irreversible limit state. For partial reversible limit state, the target index could be determined between 0 and 1.5, depending on the degree of reversibility. The study on the flexural behavior of GFRP-reinforced concrete beams has indicated that those beams could exhibit excellent reversibility of crack width if applied load is removed (He, 2003). So, the global average reliability of 0.57 is acceptable as a whole. Figures 2-3 illustrate the effects of load effect ratio, α, and FRP relative reinforcement ratio, η, on the average level. Unlike the significant effect of η on the average reliability level of deflection of GFRP-reinforced concrete beams (He and Qiu, 2007), it seems that η does not put any effect on reliability level of maximum crack width of GFRP-reinforced concrete beams. As we expected, the effect of load effect ratio, α, is obvious, especially for α is less than 1.5. The average index increases at a slowing rate as α increases. The average index would increase about 7.6% if α increases from 0.5 to 2.5, tending to converge at β=0.58.

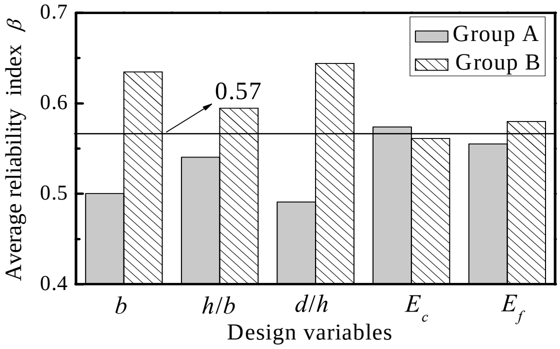

Figure 1 Effects of design variables on β

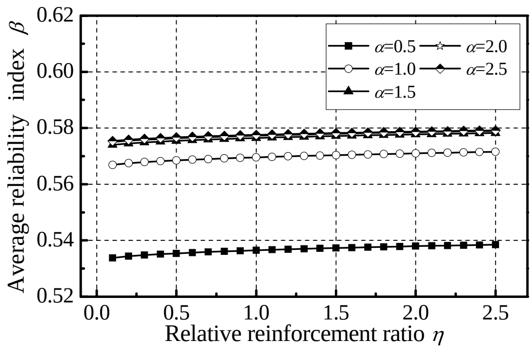

Figure 2 The relationships between β, η and α

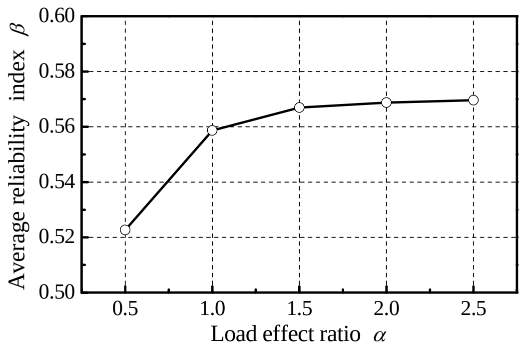

Figure 3 Effect of α on β

4 PARAMETRIC STUDY

As shown in Figure 2, the average reliability index is basically independent of relative FRP reinforcement ratio, η. So, only single value of η is selected, i.e. η=2.0 in the following parametric study of the effects of design variables. Figures 4-6 show the relationships between average reliability index and b, h/b and d/h. As b increases gradually from 150mm to 500mm, the average reliability index increases significantly and nonlinearly, exhibiting a close interrelationship between average reliability level and sectional width. The relationship between average reliability level and height-to-width ratio, h/b, can be approximated by a straight line. Figure 6 shows that average reliability index would increase slowly and nearly linearly as effective-height-to-height ratio, d/h, increases from 0.5 to 0.8, and that if d/h>0.8, the index would increase very rapidly as d/h increases up 0.95. The statistics of six values of concrete strength (Nowak & Szerszen, 2003), f_c', are selected to evaluate the statistics of the modulus of elasticity of concrete, E_c, by Eq.(5). The relationship between average reliability index and E_c is illustrated in Figure 7 from which we can observe that and the mean and the standard deviation of E_c have some effects on average reliability level. But those effects are insignificant in contrast with those of sectional design variables. In a general sense, the average index decreases as the nominal value of E_c increases.

The effect of bond-dependent coefficient, k_b, on reliability level is also obvious. For FRP bars having bond behavior inferior to steel, k_b is larger than 1.0, and for FRP bars having bond behavior superior to steel, k_b is smaller than 1.0. The average k_b values vary widely from 0.60 to 1.72, with a mean of 1.10 (ACI 440, 2006). To make further understanding about the effect of k_b on reliability level, k_b is treated as a random variable here with assumed normal probability distribution. Its mean ranges from 0.6 to 1.7 and four values of C.O.V. are also taken into consideration, i.e. δ=0.05, 0.10, 0.15 and 0.20. The analytical results are illustrated in Figure 8, showing that the mean of k_b does not put any effect on reliability level. The effect of the C.O.V. of k_b is shown to be rather significant. As the C.O.V. of k_b increases from 0.05 to 0.20, the average index decreases from 0.53 to 0.41. Since the limit for maximum crack width, $[w]$, is a constant as specified in ACI 440 guideline (ACI 440, 2006), the reliability level has no relations with [*w*] which is confirmed by Figure 9. So, from the probabilistic point of view, any value of [*w*] can be specified for the design of the serviceability state of FRP-reinforced concrete beams. Specification on the limit of maximum allowable crack width is dependent only on the consideration of durability and visible tolerance.

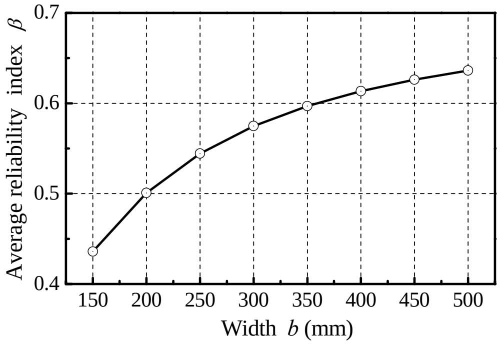

Figure 4 Effect of b on β

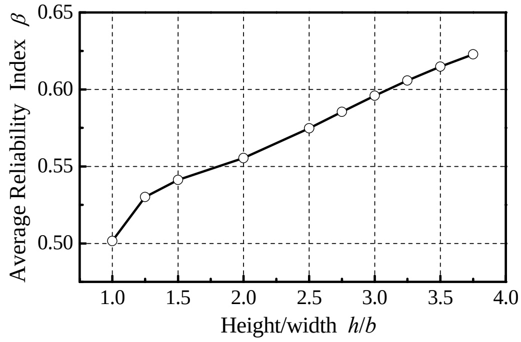

Figure 5 Effect of h/b on β

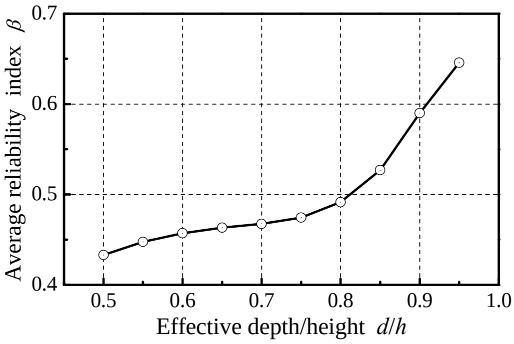

Figure 6 Effect of d/h on β

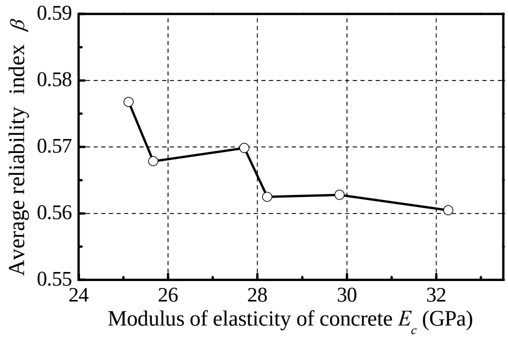

Figure 7 Effect of E_c on β

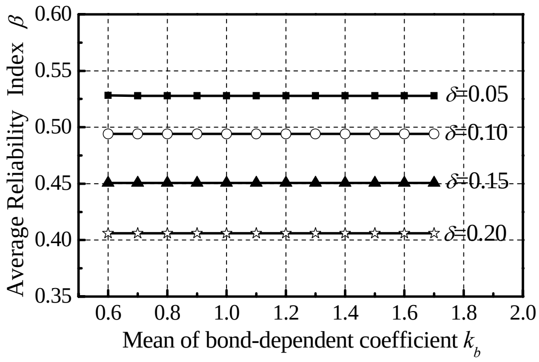

Figure 8 Effect of k_b on β

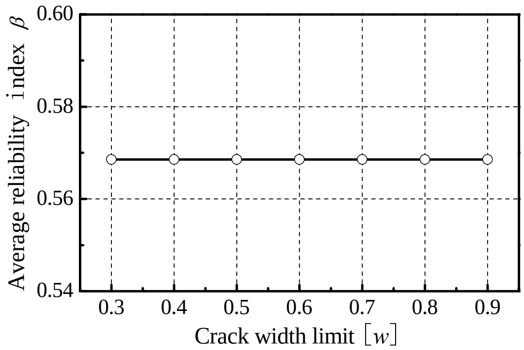

Figure 9 Effect of [w] on β

5 CONCLUSIONS

The reliability assessment indicates that the global average reliability index is 0.57, meeting the requirement of the target reliability index for serviceability ultimate state. Effective-height-to-height ratio, d/h, width, b, and height-to-width ratio, h/b, are the dominating influencing factors on reliability level. The average reliability index is basically independent of relative FRP reinforcement ratio, η. As load effect ratio, α, increases, the average reliability level also increases but a slowing rate. The parametric study on the bond-dependent coefficient, k_b, indicates that its mean does not put any influence on reliability level while its C.O.V. is rather distinct. The limit of the maximum crack width, [w], is shown to be no relations with reliability level. Specification on the limit of maximum allowable crack width is dependent only on the consideration of durability and visible tolerance.

6 ACKNOWLEDGEMENT
This research work was financially supported by the National Natural Science Foundation of China (Grant No. 50778035) and the Ministry of Education of People's Republic of China.

REFERENCES

ACI 318-05. 2005. *Building code requirements for structural concrete*. Detroit: American Concrete Institute, USA.

ACI 440.1R-06. 2006. *Guide for the design and construction of concrete reinforced with FRP bars*. Detroit: American Concrete Institute, USA.

Ellingwood B., Galambos T.V., MacGregor J.G. and Cornell C.A. 1980. *Development of a probability based load criterion for American national standard A58 building code requirements for minimum design loads in buildings and other structures*. Special Publication 577, Washington D.C.: National Bureau of Standards, USA.

Gao D. Y., Zhao J. and Benmokrane B. 2001. The characteristics of crack and deflection and its calculating method of concrete beam reinforced with glass fiber polymer bars (in Chinese). *Journal of Hydraulic Engineering*, 8: 53-58.

He Z. 2003. Damage-performance-based structural seismic design and basic mechanical properties of FRP rebars and their reinforced concrete members (in Chinese). *Postdoctoral report*, Harbin Institute of Technology, China.

He Z., Li G. and Huang Y. C. 2006. Development of reliability-based flexural design for FRP-reinforced concrete beams by Chinese background. *Pacific Science Review*, 8(1):123-131.

He Z. and Ou J. P. 2005. Probability-based design for FRP reinforced concrete structures: a critical review and research needs. *Proceedings of the 50th SAMPE Symposium*, Long Beach, USA.

He Z. and Qiu F. 2007. Reliability assessment on the serviceability of concrete reinforced with glass fiber reinforced polymer rods. *Proceedings of Asia-Pacific Conference on FRP in Structures*, Hong Kong, China.

Huang Y. C. 2005. Reliability assessment of FRP-reinforced concrete members designed by ACI and ISIS design guidelines (in Chinese). *Master thesis*, Harbin Institute of Technology, Harbin, China.

ISO/DIS 2394. 1998. General principles on reliability for structures.

Liu H. J. 2003. Experimental and theory analytical studies on concrete flexural members reinforced with FRP rebars (in Chinese). *Master thesis*, Tongji University, Shanghai, China.

Masmoudi R., Thériault M., Benmokrane B. 1998. Flexural behavior of concrete beams reinforced with deformed fiber reinforced plastic reinforcing rods. *ACI Structural Journal*, 95(6):665-676.

Nowak A. S. and Szerszen M. M. 2003. Calibration of design code for buildings (ACI 318): part I-statistical models for resistance. *ACI Structural Journal*, 100(3):377-38.

Pilakoutas K., Neocleos K. and Guadagnini M. 2002. Design philosophy issues of fiber reinforced poly mer reinforced concrete structures", *Journal of Composites for Construction*, 6(3), 154-161.

Plevris N., Triantafillou T. C., Veneziano D. 1995. Reliability of RC members strengthened with CFRP laminates. *Journal of Structural Engineering*, ASCE, 121(7):1037-1044.

Qiu F. 2007. Reliability-based design for GFRP-reinforced concrete beams (in Chinese). *Master thesis*, Northeastern University, Shenyang, China.

Rackwitz R. and Fiessler B. 1978. Structure reliability under combined random sequences. *Computers & Structures*, 9: 489-494.

Thériault M. and Benmokrane B. 1998. Effects of FRP reinforcement ratio and concrete strength on the flexural behaviour of concrete beams. *Journal of Composites for Construction*, ASCE, 2(1):7-16.

Toutanji H. A. and Saafi M. 2003. Deflection and crack width predictions of concrete beams reinforced with fiber reinforced polymer bars. *Proceedings of the 6th International Symposium on Fiber Reinforced Polymer Reinforcement for Reinforced Concrete Structures (FRPRCS-6)*, Tan K H (ed.), Singapore.

Wang H. and Belarbi A. 2005. Flexural behavior of fiber-reinforced-concrete beams reinforced with FRP rebars. *Proceedings of the 7th International Symposium on Fiber Reinforced Polymer Reinforcement for Reinforced Concrete Structures (FRPRCS-7)*, Shield C. K. *et al*. (ed.), Kansas City, USA.

Wang Z. Z. and Shi B. 2002. Research on the computational method of deflection and crack width of concrete beams reinforced with GFRP bars (in Chinese). *Industrial Construction*, 32(11): 8-10.

Xue W. C. and Kang Q. L. 1999. Experimental study on behaviors of concrete beams reinforced with fiber reinforced plastics bars (in Chinese). *Industrial Construction*, 29(12):8-10.